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Effects of chemical reaction, viscous dissipation and pressure work on MHD free convection flow in a porous medium

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ABSTRACT

The effects of chemical reaction, viscous dissipation and pressure work are included in a three-parameter perturbation analysis in magnetohydrodynamic free convection flows of Newtonian fluid-saturated porous medium. The Rosseland approximation is used to describe the radiative heat flux in the energy equation. Four different cases of flows have been studied namely an isothermal surface, a uniform surface heat flux, a plane plume and flow generated from a horizontal line energy source along a vertical adiabatic surface. Numerical results are presented for the perturbation analysis for the four boundary conditions. The obtained results are compared and a representative set is displayed graphically to illustrate the influence of the parameters on the velocity, temperature and concentration.

KEYWORDS: MHD, chemical reaction, perturbation analysis, viscous dissipation, pressure work, porous medium

1. INTRODUCTION

Natural convection in fluid saturated in porous media arises in a large number of natural sciences as well as several branches of technology. These include geophysics, soil mechanics, metal casting, ceramic engineering, the technology of paper and insulating materials. Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer processes occur simultaneously. Therefore, the porous media play a vital role in many engineering applications such as thermal insulation of buildings, energy recovery of petroleum resources, chemical reactors and nuclear waste disposals. Cheng and Minkowycz [1] presented an analysis for the natural convection flows about a heated impermeable surface embedded in fluid-saturated porous media, to model the heating of groundwater in an aquifer by a dike. Kaviany and Mittal [2] considered the first order boundary layer approach both for vertical and horizontal natural convection flows using singular perturbation analysis. Ali, *et al.* [3] studied the interaction of natural convection with the thermal radiation in a laminar boundary layer flow over an isothermal, horizontal flat plate. Sahar, *et al.* [4] studied radiative effect on natural convection flows in porous media using the effects of both first and second –order resistances due to the solid matrix on some natural convection flows in fluid–saturated porous media. Soundalgekar and Takhar [5] studied radiation effects on free convection flow of a gas past a semi-infinite flat plate. Hossain, *et al.* [6] determined the effect of radiation on natural convection flow of an optically thick viscous incompressible flow past a heated vertical porous plate with a uniform surface temperature and a uniform rate of suction where radiation is included by assuming the Rosseland diffusion approximation. Hossain and Rees [7] investigated free convection from isothermal inclined plates to horizontal plates. Yih [8] studied the effect of radiation on natural convection about a truncated cone. Hossain and Pop [9] analyzed the radiation effect on free convection flow along an inclined surface placed in a porous medium. At high temperatures thermal radiation can significantly affect the heat transfer and the temperature distribution in the boundary layer flow of participating fluid. Gorla [10], and Gorla and Pop [11] investigated the effects of radiation on mixed convection flow over vertical cylinders. Ibrahiem and Hady [12] studied mixed convection–radiation interaction in boundary layer flow over a horizontal surface. Forced convection–radiation interaction heat transfer in boundary-layer over a flat plate embedded in a porous medium was analyzed by Mansour [13].

Yih [14] studied the radiation effect on free convection over a vertical cylinder embedded in porous media. EL-Hakim and Rashad [15] used Rosseland diffusion approximation in studying the effect of radiation on free convection from a vertical cylinder embedded in a fluid-saturated porous medium. Rashad [16] studied the radiative effect on heat transfer from an arbitrarily stretching surface with non-uniform surface temperature embedded in a porous medium.

The present investigation is devoted to a study of the effects of chemical reaction on some natural convection flows of Newtonian fluid-saturated porous medium in the presence of radiation, viscous dissipation, magnetic field, permeability of porous media, pressure work and heat generation past a vertical surface. Numerical results are presented for four representative kinds of surface temperature variation, namely, an isothermal surface, a uniform heat flux surface, a plane plume and flow generated from a horizontal line energy source and a vertical adiabatic surface.

2. Analysis

We consider a steady two-dimensional natural convection hydromagnetic flow of a viscous incompressible, electrically conducting fluid past a vertical plate and incorporate the usual Boussinesq and boundary layer assumptions. The flow is taken to be in the direction of *x*-axis, and *y*-axis normal to it. The temperature of the quiescent ambient fluid, T_∞ at large values of *y*, is taken to be constant. The fluid properties are assumed to be constant and the magnetic field is normal to the direction of flow. The governing equations for the problem under consideration can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta_T\left(T - T_\infty\right) + g\beta_C\left(C - C_\infty\right) - \left(\frac{\sigma B_0{}^2}{\rho} + \frac{\nu}{k_1}\right)u, \tag{2}$$

$$u\frac{\partial T}{\partial x} + \upsilon\frac{\partial T}{\partial y} = \frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_P}\left(\frac{\partial u}{\partial y}\right)^2 - \frac{1}{\rho C_P}\frac{\partial q_r}{\partial y} + \frac{\beta_T T_f}{\rho C_P}u\frac{dP_h}{dx} + \frac{Q_0}{\rho C_P}\left(T - T_\infty\right), \tag{3}$$

$$u\frac{\partial C}{\partial x} + \upsilon\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - k_c\left(C - C_\infty\right), \tag{4}$$

where, T is the temperature, C is the concentration, σ is the electrical conductivity of the fluid, B_0 is the strength of magnetic field, ρ is the density, μ is dynamic viscosity, k_1 is the permeability of porous medium, k_c, C_P and g are the rate of chemical reaction, specific heat of the fluid and acceleration due to gravity respectively. β_T is the thermal expansion coefficient, β_c is the concentration expansion coefficient, D is the mass diffusion coefficient, Q_0 is the heat generation constant, C_∞ and T_∞ are the free stream dimensional concentration and temperature, respectively, q_r is the radiative heat flux., T_f is the film temperature. The fluid pressure consists of the hydrostatic and motion pressure: $P = P_h + P_m$.

The motion pressure is considered small compared to hydrostatic pressure and is ignored [17, 18]. For the hydrostatic pressure we have:

$$\frac{dP_h}{dx} = -\rho g\,, \tag{5}$$

Also the radiative heat flux term is simplified by using the Rosseland approximation (see Sparrow Cess [19]) as

$$q_r = -\frac{4\sigma_0}{3k^*}\frac{\partial T^4}{\partial y}, \tag{6}$$

where σ_0 and k^* are the Stefan-Boltzman constant and mean absorption coefficient, respectively. The obtained Taylor series expansion for T^4 neglecting higher order terms:

$$T^4 = 4T_\infty^3 T - 3T_\infty^4\,, \tag{7}$$

Using Eqs. (5), (6) and (7) in energy equation (3) we obtain

$$u\frac{\partial T}{\partial x} + \upsilon\frac{\partial T}{\partial y} = \frac{\nu}{\Pr}\left[1+\frac{4}{3}R\right]\frac{\partial^2 T}{\partial y^2} - \frac{\beta_T T_f}{C_P}u\,g + \frac{\mu}{\rho C_P}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{Q_0}{\rho C_P}\left(T - T_\infty\right), \tag{8}$$

where $\Pr = \frac{\rho\,\nu\,C_P}{k}$ is the Prandtl number, $R = \frac{4\sigma T_\infty^3}{k\,k^*}$ is the radiation parameter and ν is the kinematic viscosity.

The following transformation can be introduced:

$$\eta(x,y) = y\,b(x),$$

$$\psi(x,y) = \nu\,h(x)\,f(\eta,x),$$

$$h(x) = 4xb(x) = 4\left(\frac{g\beta_T x^3 (T_0 - T_\infty)_0}{4\nu^2}\right)^{\frac{1}{4}} = 4\left(\frac{Gr_x}{4}\right)^{\frac{1}{4}},$$

$$\theta(\eta,x) = \frac{T - T_\infty}{\left(T_0 - T_\infty\right)_0}, \quad \left(T_0 - T_\infty\right)_0 = d(x) = sx^n,$$

$$\phi(\eta,x) = \frac{C - C_\infty}{\left(C_0 - C_\infty\right)_0}, \quad \left(C_0 - C_\infty\right)_0 = d_1(x) = s_1 x^n, \tag{9}$$

here b, d and h are linear function in x and $Gr_x = \frac{g\beta_T x^3 (T_0 - T_\infty)_0}{\nu^2}$ is local Grashof number and $\left(T_0 - T_\infty\right)_0$ is the downstream temperature differences (along the x axis). Expansions for the stream function $f(\eta,x)$, temperature function $\theta(\eta,x)$ and concentration function $\phi(\eta,x)$ are:

$$f(\eta,x) = f_0(\eta) + \varepsilon(x)\,f_1(\eta) + \lambda_1(x)\,f_2(\eta) + \lambda(x)\,f_3(x), \tag{10}$$

$$\theta(\eta,x) = \theta_0(\eta) + \varepsilon(x)\,\theta_1(\eta) + \lambda_1(x)\,\theta_2(\eta) + \lambda(x)\theta_3(x), \tag{11}$$

$$\phi(\eta,x) = \phi_0(\eta) + \varepsilon(x)\,\phi_1(\eta) + \lambda_1(x)\,\phi_2(\eta) + \lambda(x)\phi_3(x), \tag{12}$$

where the permeability of the porous medium, magnetic field, viscous dissipation, pressure work, chemical reaction and heat generation terms, are related as:

$$\varepsilon(x) = \frac{4g\beta_T}{C_P}x, \tag{13}$$

$$\lambda_1(x) = \frac{4\,g\,\beta_T T_f\,x^{1-n}}{S\,C_P} \tag{14}$$

$$\lambda(x) = \frac{2k_c}{\left(g\beta_T s x^{n-1}\right)^{1/2}}. \tag{15}$$

The choice of $\varepsilon(x)$ is the same as Gebhart [20] from the viscous dissipation effect. The quantities of $\lambda_1(x)$ and $\lambda(x)$ are due to the pressure work effect, permeability of porous medium, heat generation and chemical reaction effects. Note that $\lambda_1(x)$ and $\lambda(x)$ are constant when $n = 1$. $\varepsilon(x)$, $\lambda_1(x)$ and $\lambda(x)$ are treated as prescribable parameters. This has been done to study the effect of each parameter on the velocity, temperature and concentration fields. Greater accuracy for specific circumstances may be obtained by retaining higher order terms in (10)-(12).

Substituting (10)-(12) into Eqs. (2), (4) and (8) with the generalization in (9), the equations for $f_0, \theta_0, \phi_0, f_1, \theta_1, \phi_1, f_2, \theta_2, \phi_2, f_3, \theta_3, \phi_3$ are determined for any value of n as:

$$f_0''' + \theta_0 + N\phi_0 - (2n+2)\, f_0'^2 + (n+3)\, f_0\, f_0'' = 0, \tag{16}$$

$$\left(1+\frac{4}{3}R\right)\theta_0'' + \Pr\left(-4n\, f_0'\theta_0 + (n+3)\, f_0\, \theta_0'\right) = 0, \tag{17}$$

$$\phi_0'' - Sc\left(4nf_0'\phi_0 - (n+3)\, f_0\phi_0'\right) = 0, \tag{18}$$

$$f_1''' + \theta_1 + N\phi_1 - 4(n+2)\, f_0' f_1' + (n+3)\, f_0\, f_1'' + (n+7)\, f_1\, f_0'' = 0, \tag{19}$$

$$\left(1+\frac{4}{3}R\right)\theta_1'' + pr\left(f_0'^2 - 4(n+1)\, f_0'\theta_1 - 4n\, f_1'\theta_0 + (n+7)\, f_1\theta_0' + (n+3)\, f_0\theta_1'\right) = 0, \tag{20}$$

$$\phi_1'' - Sc\left(4(n-1)\, f_0'\phi_1 + 4nf_1'\phi_0 - (n-1)\, f_1\,\phi_0' - (n+3)\, f_0\phi_1'\right) = 0, \tag{21}$$

$$f_2''' + \theta_2 + N\phi_2 - 8\, f_0'\, f_2' - (3n-7)\, f_0''\, f_2 + (n+3)\, f_0\, f_2'' + (5-n)\, f_2\, f_0'' = 0, \tag{22}$$

$$\left(1+\frac{4}{3}R\right)\theta_2'' + pr\left(-f_0' - 4f_0'\,\theta_2 - 4n\, f_2'\theta_0 - (3n-7)\, f_2\,\theta_0' + (n+3)\, f_0\,\theta_2'\right) = 0, \tag{23}$$

$$\phi_2'' - Sc\left(4(2n-1)\, f_0'\,\phi_2 + 4nf_2'\,\phi_0 - (5n-1)\, f_2\,\phi_0' - (n+3)\, f_0\,\phi_2'\right) = 0, \tag{24}$$

$$f_3''' + \theta_3 + N\phi_3 - (M+K)\frac{1}{\gamma}\, f_0' - 2(3n+1)\, f_0'\, f_3' - (n+3)\, f_0\, f_3'' - (3n+1)\, f_3\, f_0'' = 0, \tag{25}$$

$$\left(1+\frac{4}{3}R\right)\theta_3'' + pr\left(-4nf_3'\,\theta_0 - 2(3n-1)\, f_0'\,\theta_3 + (3n+1)\, f_3\,\theta_0' + (n+3)\, f_0\,\theta_3' + \frac{Q}{\gamma}\theta_0\right) = 0, \tag{26}$$

$$\phi_3'' - Sc\left(\phi_0 - 4nf_3'\,\phi_0 - (5n-2)\, f_0'\,\phi_3 - (n+3)\, f_0\,\phi_3' + (n-5)\, f_3\,\phi_0'\right) = 0, \tag{27}$$

where $M = \frac{\sigma B_0^2 \nu}{\rho U^2}$, $K = \frac{\nu^2}{k_1 U^2}$ and $\gamma = \frac{k_c \nu}{U^2}$ are magnetic field parameter, permeability parameter and chemical reaction parameter, respectively. $Q = \frac{Q_0 \nu}{\rho C_P U^2}$ is the heat generation parameter. $Sc = \frac{\nu}{D}$ is the Schmidt number and $N = \frac{\beta_C (C_0 - C_\infty)_0}{\beta_T (T_0 - T_\infty)_0} = \frac{Gr_c}{Gr_x}$, $Gr_c = \frac{g\beta_c x^3 (C_0 - C_\infty)_0}{\nu^2}$ are buoyancy ratio and modified Grashof number, respectively. U is reference velocity. The boundary conditions for the zero-order equations are taken to be those that would arise in the absence of the radiation, viscous dissipation, and permeability of porous media, magnetic field, heat generation, pressure work and chemical reaction effects. The boundary conditions for the first-order terms are then found by imposing reasonable requirements on the velocity, temperature and the concentration functions $f(\eta, x), \theta(\eta, x)$, $\phi(\eta, x)$ and their derivatives at $\eta = 0$ and as $\eta \to \infty$:

(a) Isothermal surface with horizontal leading edge $n = 0$

$$f_0'(0) = f_1'(0) = f_2'(0) = f_3'(0) = 0,$$
$$f_0(0) = f_1(0) = f_2(0) = f_3(0) = 0,$$

$$1 - \theta_0(0) = \theta_1(0) = \theta_2(0) = \theta_3(0)$$
$$= 1 - \phi_0(0) = \phi_1(0) = \phi_2(0) = \phi_3(0) = 0,$$

$$f_0'(\infty) = f_1'(\infty) = f_2'(\infty) = f_3'(\infty) 0,$$

$$\theta_0(\infty)=\theta_1(\infty)=\theta_2(\infty)=\theta_3(\infty)=0,$$
$$\phi_0(\infty)=\phi_1(\infty)=\phi_2(\infty)=\phi_3(\infty)=0.$$

(b) Uniform-flux surface with horizontal leading edge, $n=0.2$

$$f_0'(0)=f_1'(0)=f_2'(0)=f_3'(0)=0,$$
$$f_0(0)=f_1(0)=f_2(0)=f_3(0)=0,$$

$$1-\theta_0(0)=\theta_1'(0)=\theta_2'(0)=\theta_3'(0)=0,$$
$$1-\phi_0(0)=\phi_1'(0)=\phi_2'(0)=\phi_3'=0,$$

$$f_0'(\infty)=f_1'(\infty)=f_2'(\infty)=f_3'(\infty)=0,$$

$$\theta_0(\infty)=\theta_1(\infty)=\theta_2(\infty)=\theta_3(\infty)$$
$$=\phi_0(\infty)=\phi_1(\infty)=\phi_2(\infty)=\phi_3(\infty)=0.$$

(c) Unbounded plane plume, rising from horizontal thermal source at $x=0$, $n=-0.6$

$$f_0(0)=f_1(0)=f_2(0)=f_3(0)$$
$$=f_0''(0)=f_1''(0)=f_2''(0)=f_3''(0)=0,$$

$$1-\theta_0(0)=\theta_0'(0)=\theta_1'(0)$$
$$=\theta_2'(0)=\theta_3'(0)=0,$$

$$1-\phi_0(0)=\phi_0'(0)=\phi_1'(0)$$
$$=\phi_2'(0)=\phi_3'(0)=0,$$

$$f_0'(\infty)=f_1'(\infty)=f_2'(\infty)=f_3'(\infty)=0,$$

$$\theta_1(\infty)=\theta_2(\infty)=\theta_3(\infty)$$
$$=\phi_1(\infty)=\phi_2(\infty)=\phi_3(\infty)=0.$$

(d) An adiabatic surface with a concentrated heat source along the horizontal leading edge, $n=-0.6$

$$f_0'(0)=f_1'(0)=f_2'(0)=f_3'(0)=f_0(0)$$
$$=f_1(0)=f_2(0)=f_3(0)=0,$$

$$1-\theta_0(0)=\theta_0'(0)=\theta_1'(0)$$
$$=\theta_2'(0)=\theta_3'(0)=0$$

$$1-\phi_0(0)=\phi_0'(0)=\phi_1'(0)$$
$$=\phi_2'(0)=\phi_3'(0)=0,$$

$$f_0'(\infty)=f_1'(\infty)=f_2'(\infty)=f_3'(\infty)=0,$$

$$\theta_1(\infty)=\theta_2(\infty)=\theta_3(\infty)=\phi_1(\infty)=\phi_2(\infty)=\phi_3(\infty)=0.$$

In the previous equations, the primes indicate differentiations with respect to η only, the value of n in (9), $(T_0-T_\infty)_0=d(x)=sx^n$ and $(C_0-C_\infty)_0=d_1(x)=s_1x^n$ depends only on the zeroth order solution. For the isothermal condition we have $n=0$ and, therefore $(T_0-T_\infty)_0$ is given. The values of n for the other three flow conditions are determined by calculating the value of $Q_0(x)$, the total heat convected in the flow at downstream location x, considering only the zeroth-terms. The energy equation (8) in the absence of viscous dissipation, radiation, pressure work and heat generation terms is integrated at a given x to lead

$$Q_0(x)=\int_0^\infty \rho C_p u_0 (T-T_\infty)_0 dy=\int_0^x q_w'' dx,$$

where q_w'' is the surface heat flux and the subscript (0) emphasizes that the viscous dissipation, pressure work, radiation and heat generation effects are not being considered.

Using generalization in (9), (10), (11) and (12) we get:

$$Q_0(x)=\int_0^\infty \rho C_p u_0 (T-T_\infty)_0 dy$$
$$=\rho\nu C_p h(x) d(x)\int_0^\infty \theta_0 f_0' d\eta \propto x^{\frac{(3+5n)}{4}}, \tag{28}$$

This must increase linearly with x for the uniform heat flux surface condition (b) and is independent of x for the adiabatic flows, (c) and a plane plume flows, (d). Therefore, we have: $n_a=0$, $n_b=0.2$, $n_c=n_d=-0.6$.

Even in the plume flow, the total convected energy does change downstream at $x=0$, including the zeroth and first order terms, $Q(x)$ is, in general

$$Q(x)=\rho\nu C_P h(x)d(x)\left[\int_0^\infty \theta_0 f_0' d\eta+\varepsilon(x)\int_0^\infty (\theta_0 f_1'+\theta_1 f_0')d\eta\right.$$
$$\left.+\lambda_1(x)\int_0^\infty (\theta_0 f_2'+\theta_2 f_0')d\eta+\lambda(x)\int_0^\infty (\theta_0 f_3'+\theta_3 f_0')d\eta\right], \tag{29}$$

Also we can write $Q_c(x)$, the total mass diffusion convected in the flow at any downstream location *x* as:

$$Q_c(x)=\int_0^\infty \rho\left(C-C_\infty\right)u\,dy$$
$$=\rho\nu\, h(x)\, d_1(x)\int_0^\infty \phi f'\, d\eta \propto x^{(3+5n)/4}, \tag{30}$$

Under the same conditions as total heat convected we can write $Q_c(x)$ in the form:

$$Q_c(x)=\rho\nu h(x)d_1(x)\left[\int_0^\infty \varphi_0 f_0' d\eta+\varepsilon(x)\int_0^\infty (\varphi_0 f_1'+\varphi_1 f_0')d\eta\right.$$
$$\left.+\lambda_1(x)\int_0^\infty (\varphi_0 f_2'+\varphi_2 f_0')d\eta+\lambda(x)\int_0^\infty (\varphi_0 f_3'+\varphi_3 f_0')d\eta\right], \tag{31}$$

The local total downstream mass flow rate per unit width is given by:

$$m=\int_0^\infty \rho u\, dy=\rho\nu\, h(x)\int_0^\infty f'\, d\eta=\rho\nu\, h(x)\left[f_0(\infty)\right.$$
$$\left.+\varepsilon(x)f_1(\infty)+\lambda_1(x)f_2(\infty)+\lambda(x)f_3(\infty)\right], \tag{32}$$

The total $x-$ direction momentum flux is given by:

$$M(x)=\int \rho u^2 dy=\rho\nu^2(h(x))^2 b(x)\int f'^2 d\eta$$
$$=\rho\nu^2(h(x))^2 b(x)\left[I_{M_0}+\varepsilon(x)I_{M_1}\right.$$
$$\left.+\lambda_1(x)I_{M_2}+\lambda(x)I_{M_3}\right], \tag{33}$$

where

$$I_{M_0}=\int_0^\infty f_0'^2 d\eta,\quad I_{M_1}=\int_0^\infty 2f_0' f_1' d\eta,$$

$$I_{M_2}=\int_0^\infty 2f_0' f_2' d\eta \text{ and } I_{M_3}=\int_0^\infty 2f_0' f_3' d\eta.$$

The wall shear stress may be written as

$$\tau_w=\left[\mu\frac{\partial u}{\partial y}\right]_{y=0}, \tag{34}$$

For the surface conditions in (a-d) above, the shear stress at the surface, retaining terms up to first-order, is given by

$$\tau_w=\rho\nu^2 h b^2[f_0''(0)+\varepsilon(x) f_1''(0)$$
$$+\lambda_1(x) f_2''(0)+\lambda(x) f_3''(0)], \tag{35}$$

Also the surface heat flux, $q_w(x)$ and the local Nusselt number Nu_x determined as:

$$q_w=-k\left.\frac{\partial T}{\partial y}\right|_{y=0}-\frac{4\sigma_0}{3k^*}\left(\frac{\partial T^4}{\partial y}\right)_{y=0}$$
$$=\left[-\theta'(0)\right](T_0-T_\infty)_0\frac{k}{x}\left[\frac{Gr_x}{4}\right]^{\frac{1}{4}}\times\left(1+\frac{4}{3}R\right), \tag{36}$$

$$Nu_x=\frac{H\,x}{k}=\frac{q_w''}{(T_0-T_\infty)}\left(\frac{x}{k}\right)$$
$$=-\frac{\theta'(0)}{[\theta(0)]^{5/4}}\frac{\left(Gr_x'\right)^{1/4}}{\sqrt{2}}\left(1+\frac{4}{3}R\right), \tag{37}$$

where *k* and *H* are the thermal conductivity of the fluid and local heat transfer coefficient, respectively, and

$$\theta(0)=\theta_0(0)+\varepsilon(x)\theta_1(0)+\lambda_1(x)\theta_2(0)$$
$$+\lambda(x)\theta_3(0)+.....=\frac{T-T_\infty}{(T_0-T_\infty)_0}, \tag{38}$$

$$Gr_x'=Gr_x\theta(0), \tag{39}$$

$$S'=\frac{Nu_x\sqrt{2}}{(Gr_{x,t}')^{1/4}}=-\frac{\theta'(0)}{[\theta(0)]^{5/4}}, \tag{40}$$

where S' the heat transfer parameter.

The surface mass flux m_w and local Sherwood number Sh_x are determined as:

$$m_w = -\rho D \left.\frac{\partial C}{\partial y}\right|_{y=0}$$
$$= \left[-\phi'(0)\right](C_0 - C_\infty)_0 \frac{\rho D}{x}\left(\frac{Gr_x}{4}\right)^{1/4}, \quad (41)$$

$$Sh_x = \frac{m_w}{(C_0 - C_\infty)_0}\left(\frac{x}{\rho D}\right) = -\frac{\phi'(0)}{[\phi(0)]^{5/4}}\frac{(Gr_x'/N)^{1/4}}{\sqrt{2}} \quad (42)$$

$$S_1' = \frac{Sh_x\sqrt{2}}{(Gr_x'/N)^{1/4}} = -\frac{\phi'(0)}{[\phi(0)]^{5/4}}, \quad (43)$$

where S_1' is the mass transfer parameter and the corresponding Grashof number for mass diffusion Gr_x is related to the actual physical local Grashof number for mass diffusion Gr_x' by

$$Gr_x' = Gr_x\,\phi(0), \quad (44)$$

where

$$\phi(0) = \phi_0(0) + \varepsilon(x)\phi_1(0) + \lambda_1(x)\phi_2(0)$$
$$+\lambda(x)\phi_3(0) + ... = \frac{C - C_\infty}{(C_0 - C_\infty)_0}. \quad (45)$$

3. RESULTS AND DISCUSSION

As a result of the numerical calculation, the dimensionless velocity, temperature and concentration distribution for the flow are obtained from Eqs. (16)- (27) and are displayed in Figures 1-11 for different values of R, Pr, Sc and N. Also, we compared the numerical results of the shear stress at the surface $f_0''(0)$, and the rate of heat transfer $-\theta_0'(0)$ is shown in Table 1 for the values of n and with out the chemical reaction, pressure work, heat generation, magnetic fields, permeability of porous media, radiation and viscous dissipation with those obtained by Rashad [21]. In Figures 1 and 2, we studied the effects of buoyancy ratio with two values of Prandtl number Pr = 0.71,3 on the velocity and temperature profiles for isothermal surface. We can see the velocity increases with an increase of the buoyancy ratio parameter and have the largest values at Pr = 0.71, while the temperature decreases with an increase on the buoyancy ratio parameter and have the largest values at Pr = 0.71. Figs. 3-5 show the effects of Schmidt number on the velocity, temperature and concentration profiles for the prescribed surface heat flux case. It is noted that as Schmidt number increases, the velocity and concentration decrease whereas the temperature increases. Figs. 6-8 show the effects of buoyancy ratio on the velocity, temperature and concentration profiles for plane plume surface. We can see that as the buoyancy ratio parameter increases, the velocity

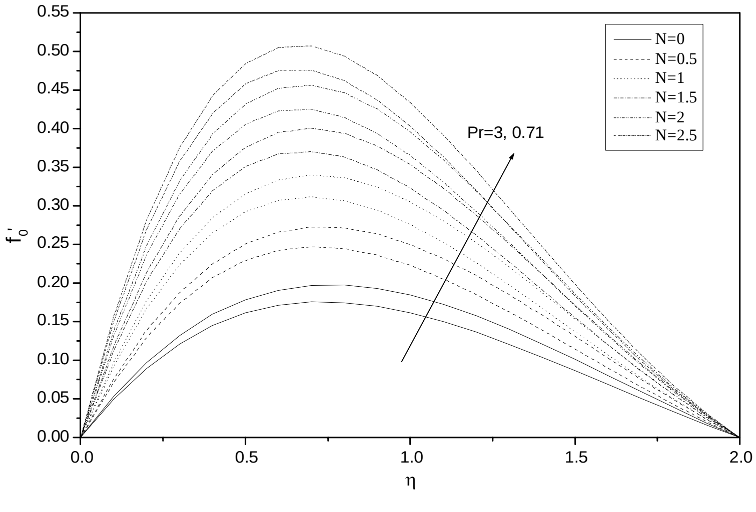

Fig. 1. The effects of buoyancy ratio on the velocity profiles for isothermal surface.

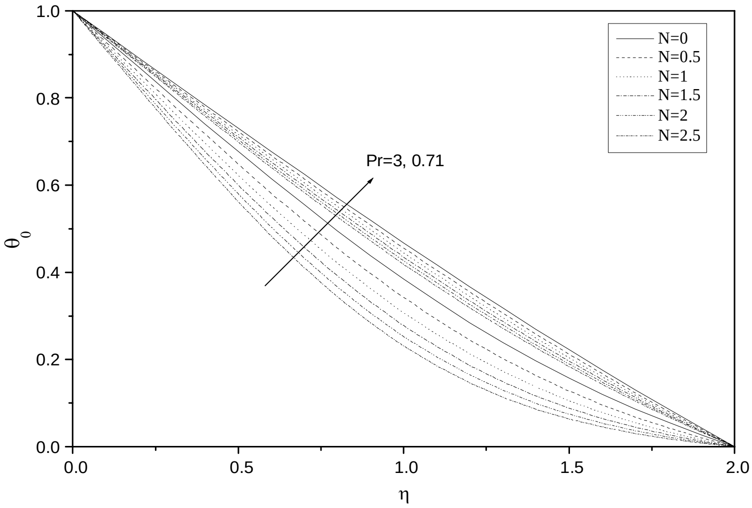

Fig. 2. The effects of buoyancy ratio on the temperature profiles for isothermal surface.

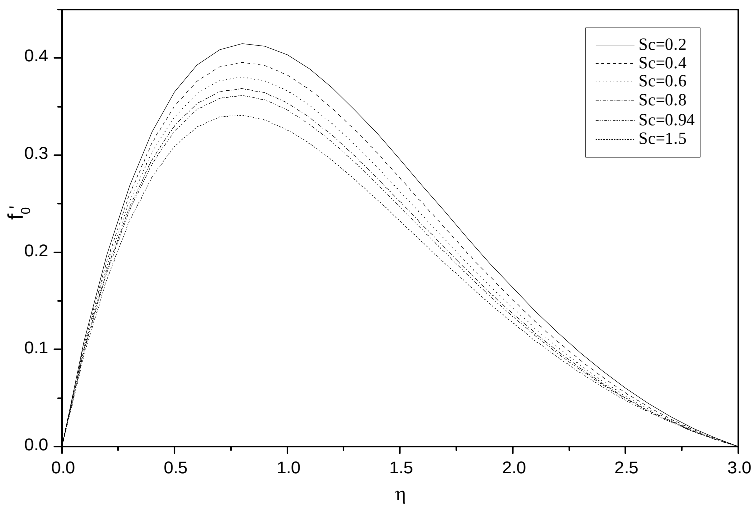

Fig. 3. The effects of Schmidt number on the velocity profiles for heat flux surface.

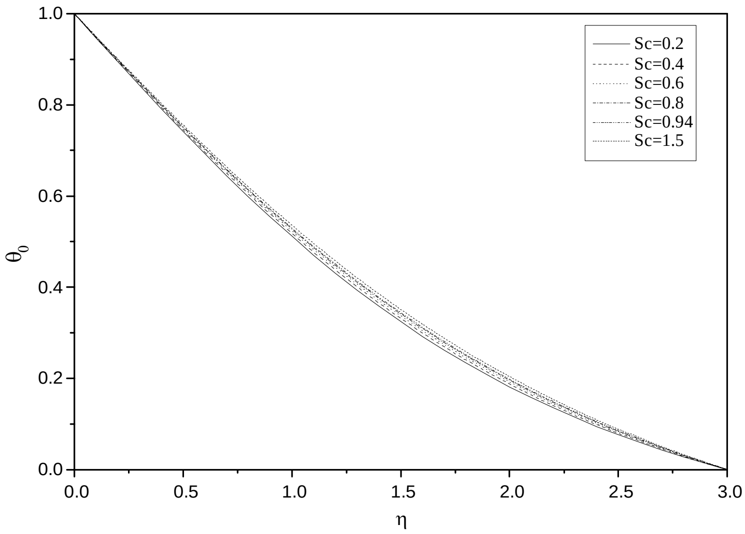

Fig. 4. The effects of Schmidt number on the temperature profiles for heat flux surface.

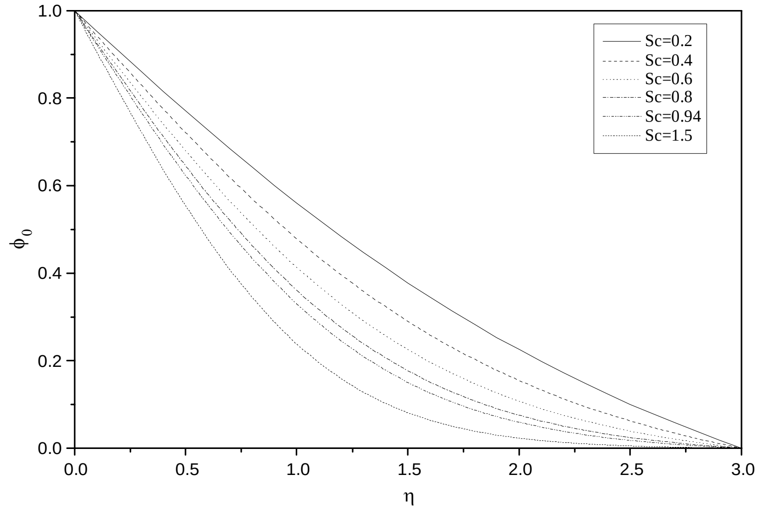

Fig. 5. The effects of Schmidt number on the concentration profiles for heat flux surface.

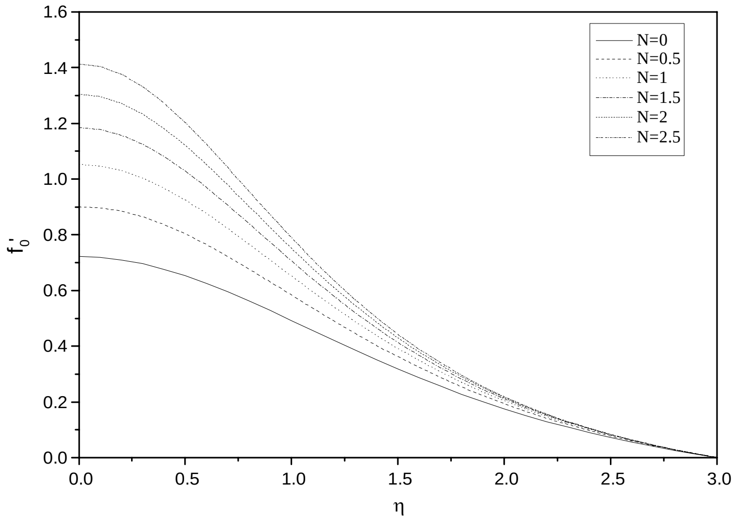

Fig. 6. The effects of buoyancy ratio on the velocity profiles for plane plume surface.

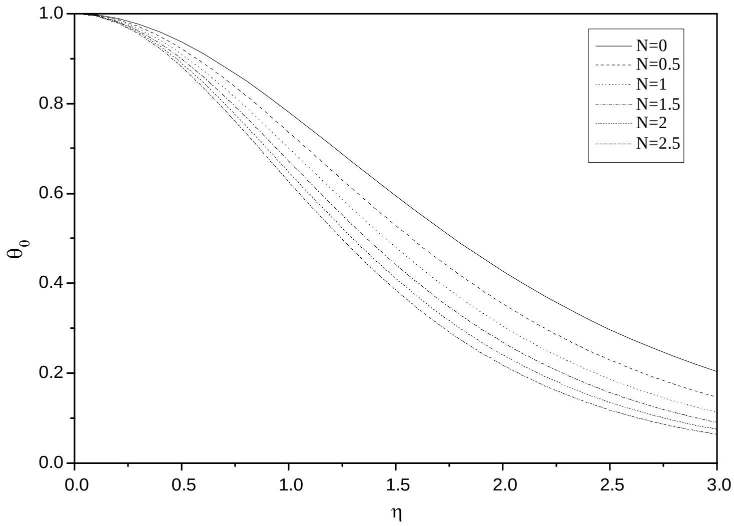

Fig. 7. The effects of buoyancy ratio on the temperature profiles for plane plume surface.

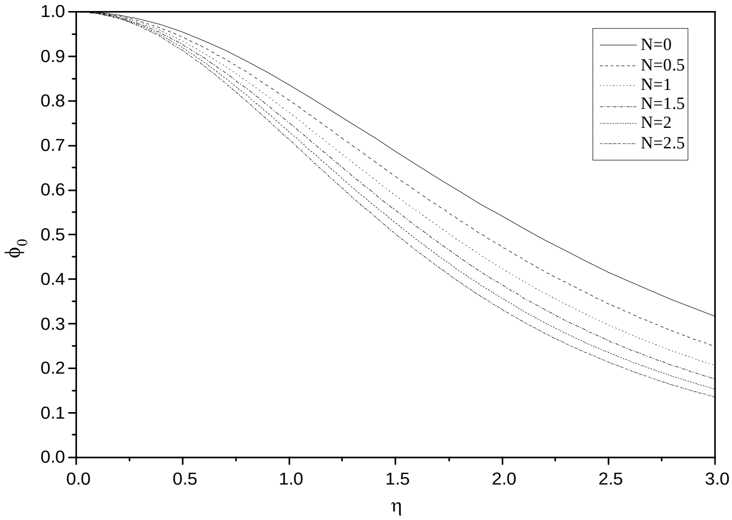

Fig. 8. The effects of buoyancy ratio on concentration profiles for plane plume surface.

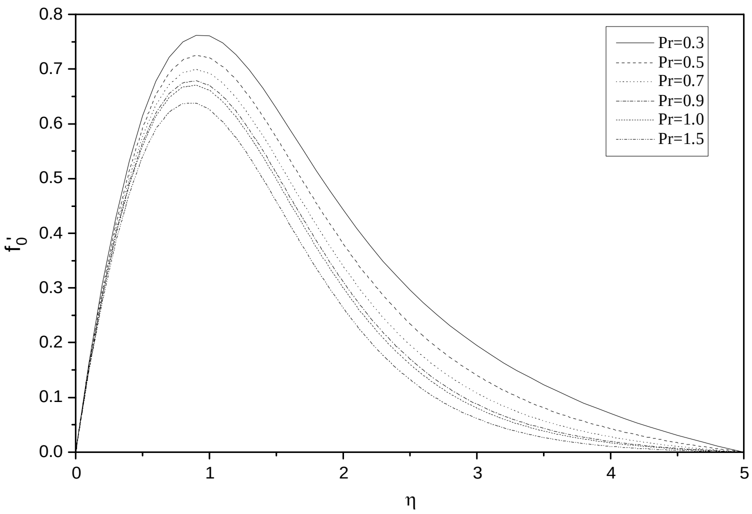

Fig. 9. The effects of Prandtl number on the velocity profiles for an adiabatic surface.

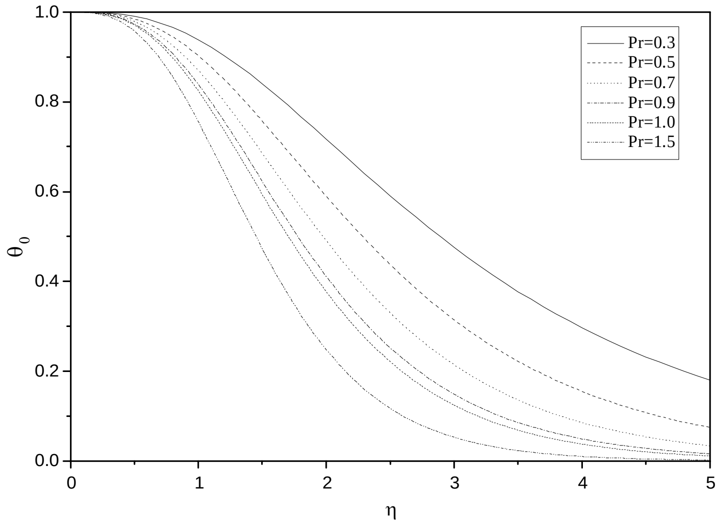

Fig. 10. The effects of Prandtl number on temperature profiles for an adiabatic surface.

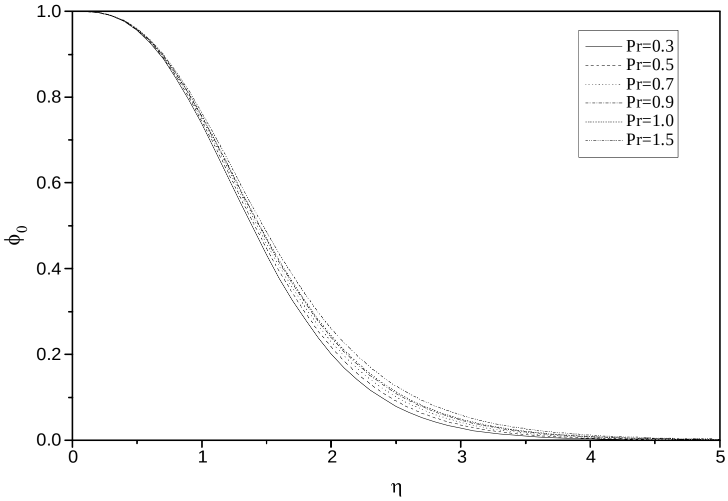

Fig. 11. The effects of Prandtl number on concentration profiles for an adiabatic surface.

Table. 1. Numerical values of f_0'' and $-\theta_0'(0)$ with $\Pr = .733$ and $N = 1$ without magnetic field, permeability of the porous medium, heat generation, pressure work, chemical reaction and viscous dissipation effects with those obtained by Rashad [21].

R	Rashad [21]		Present results	
	$f_0''(0)$	$-\theta_0'(0)$	$f_0''(0)$	$-\theta_0'(0)$
Case (a) n=0.0: isothermal surface with horizontal leading edge.				
0.0	0.66742	0.50987	0.64328	0.52961
1	0.73320	0.38803	0.69092	0.42981
2	0.75813	0.34384	0.70697	0.39718
Case (b) n=0.2: constant flux surface with horizontal leading edge.				
0.0	0.63235	0.57955	0.6127	0.59502
1	0.69872	0.43598	0.6639	0.46989
2	0.72536	0.38054	0.6823	0.42601

increases and the temperature and concentration decrease. Figs. 9-11 show the effects of Prandtl number on the velocity, temperature and concentration profiles for an adiabatic surface. It is observed that as Prandtl number increases, the velocity and temperature decrease and the concentration increases.

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